

**2019 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**

**SECTION II, Part A**

**Time—30 minutes**

**Number of questions—2**

**A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.**

1. Fish enter a lake at a rate modeled by the function  $E$  given by  $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$ . Fish leave the lake at a rate modeled by the function  $L$  given by  $L(t) = 4 + 2^{0.1t^2}$ . Both  $E(t)$  and  $L(t)$  are measured in fish per hour, and  $t$  is measured in hours since midnight ( $t = 0$ ).
- (a) How many fish enter the lake over the 5-hour period from midnight ( $t = 0$ ) to 5 A.M. ( $t = 5$ )? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ( $t = 0$ ) to 5 A.M. ( $t = 5$ )?
- (c) At what time  $t$ , for  $0 \leq t \leq 8$ , is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ( $t = 5$ )? Explain your reasoning.
-

**2019 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

$t$ (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle,  $P$ , moving along the  $x$ -axis is given by the differentiable function  $v_P$ , where  $v_P(t)$  is measured in meters per hour and  $t$  is measured in hours. Selected values of  $v_P(t)$  are shown in the table above. Particle  $P$  is at the origin at time  $t = 0$ .
- (a) Justify why there must be at least one time  $t$ , for  $0.3 \leq t \leq 2.8$ , at which  $v_P'(t)$ , the acceleration of particle  $P$ , equals 0 meters per hour per hour.
- (b) Use a trapezoidal sum with the three subintervals  $[0, 0.3]$ ,  $[0.3, 1.7]$ , and  $[1.7, 2.8]$  to approximate the value of  $\int_0^{2.8} v_P(t) dt$ .
- (c) A second particle,  $Q$ , also moves along the  $x$ -axis so that its velocity for  $0 \leq t \leq 4$  is given by  $v_Q(t) = 45\sqrt{t} \cos(0.063t^2)$  meters per hour. Find the time interval during which the velocity of particle  $Q$  is at least 60 meters per hour. Find the distance traveled by particle  $Q$  during the interval when the velocity of particle  $Q$  is at least 60 meters per hour.
- (d) At time  $t = 0$ , particle  $Q$  is at position  $x = -90$ . Using the result from part (b) and the function  $v_Q$  from part (c), approximate the distance between particles  $P$  and  $Q$  at time  $t = 2.8$ .
- 

**END OF PART A OF SECTION II**