Question 1

(a) $\int_0^5 E(t)\,dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$E(t) - L(t) = 0 \Rightarrow t = 6.20356$

$E(t) - L(t) > 0$ for $0 \le t < 6.20356,$ and $E(t) - L(t) < 0$ for $6.20356 < t \le 8.$ Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$A(t) = \int_0^t (E(s) - L(s))\,ds$

$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

$3: \begin{cases} 1: \text{sets } E(t) - L(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0,$ the rate of change in the number of fish is decreasing at time $t = 5.$

$2: \begin{cases} 1: \text{considers } E'(5) \text{ and } L'(5) \\ 1: \text{answer with explanation} \end{cases}$

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Question 2

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.

$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.

Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \quad \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \quad \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \quad \text{Extreme Value Theorem} \end{cases}$$

(b)

$$\begin{aligned} \int_0^{2.8} v_P(t)\,dt &\approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) \\ &\quad + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right) \\ &= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right) \\ &= 40.75 \end{aligned}$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$

$v_Q(t) \ge 60$ for $A \le t \le B$

$$\int_A^B \left|v_Q(t)\right|\,dt = 106.108754$$

The distance traveled by particle Q during the interval $A \le t \le B$ is 106.109 (or 106.108) meters.

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

(d) From part (b), the position of particle P at time $t = 2.8$ is

$$x_P(2.8) = \int_0^{2.8} v_P(t)\,dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t)\,dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

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